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Fractional diffusion equation with double and triple Laplace Adomian decomposition methods

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Abstract.

This paper aims to present an analytical and approximation method to get the solution of the space-time fractional diffusion equation. This suggested method is based on a combination of the double and triple Laplace transforms with the Adomain decomposition method. The presented methodology is tested on illustrative examples and the results show that it is a simple, efficient, and reliable method.

Keywords. Double Laplace transform, Triple Laplace transform, Adomain decomposition method, fractional diffusion equation, Mittag-Leffler function.

1 Introduction

Fractional calculus is a useful mathematical tool to handle applications in the area of science and engineering. The inception of the fractional calculus extends to the time integer calculus was known. It has become omnipresent in various fields such as bioengineering [1, 2], agriculture [3], viscoelasticity [4], filters [5, 6, 7], control theory [8], electronics [9], and circuits [10]. In the latter decades, Fractional Order Models FOM attracted the researchers' attention, because of being accurate and compact compared to their equivalent integer-order models [11]. Moreover, the historical dependency is one of the most important advantages of FOM which means that the next output of the model depends not only on its current state but also on all its previous states.

Recently, there has been a great interest to apply innovative methods to solve the fractional diffusion equations, due to its great importance in modeling turbulent flow, chaotic dynamics of the classical fusty system, groundwater contaminant transfer and other applications in physics, biology, chemistry and many engineering applications [12]. These methods include but not limited to: The variational iteration method (VIM) [13], the Adomian decomposition method (ADM) [14-15], Perturbation-iteration algorithm (PIA) [16] and Residual power series method (RPSM) [17].

The Laplace transform (LT) is used to solve differential equations. The cardinal idea of the LT is that it converts a differential equation into an algebraic equation, which can be solved more easily. The double and triple LT are considered as an extensive form of the original version. The double Laplace decomposition [18] and triple Laplace Adomain decomposition methods [19] have been applied on singular and coupled Burgers' equations in one and two dimensional respectively.

The organization of this paper is summarized in the following: in section 2, the basic Definitions of the fractional derivative operator and the double and triple Laplace transforms. In section 3, we solve the fractional diffusion equation in one dimension by double Laplace Adomian decomposition method using Caputo's fractional derivative with illustrative examples. The solution of the two-dimensional fractional diffusion equation has been obtained using the triple Laplace Adomian decomposition method in section 4. Finally, the conclusion is given in section 5.

2 Mathematical Definitions

2. 1 Definition The Caputo time-fractional derivative operator of order $\alpha > 0$ is defined by [20, 21]

$$D_t^{\alpha} v(r,t) = \begin{cases} \dfrac{\partial^m v(r,t)}{\partial t^m}, & for\ m = \alpha \in N; \\ \dfrac{1}{\Gamma(m-\alpha)} \displaystyle\int_0^t (t-\tau)^{m-\alpha-1} \frac{\partial^m v(r,\tau)}{\partial \tau^m} d\tau, & m-1 < \alpha < m. \end{cases}$$

2. 2 Definition The double and triple Laplace transform are defined as [18, 22]

$$L_x L_t[v(x,t)] = \iint_0^{\infty} e^{-px-st} v(x,t)\, dtdx,$$

$$L_x L_y L_t[v\,(x,y,t)] = \iiint_0^{\infty} e^{-px-qy-st} v(x,y,t) dtdydx,$$

where $x, y, t > 0$, and the variables p, q and s are Laplace variables.

The double and triple Laplace transform for the first and second order partial derivatives are given by

$$L_x L_t[\tfrac{\partial v(x,t)}{\partial x}] = pV(p,s) - V(0,s),$$

$$L_x L_y L_t[v_x\,(x,y,t)] = pV(p,q,s) - V(0,q,s),$$

$$L_x L_y L_t[v_t\,(x,y,t)] = sV(p,q,s) - V(p,q,0),$$

$$L_xL_t[\frac{\partial^2 v(x,t)}{\partial x^2}] = p^2V(p,s) - p\,V(0,s) - \frac{\partial V(0,s)}{\partial x},$$

$$L_xL_t[\frac{\partial^2 v(x,t)}{\partial t^2}] = s^2V(p,s) - s\,V(p,0) - \frac{\partial V(p,0)}{\partial t},$$

$$L_xL_yL_t[v_{xx}\,(x,y,t)] = p^2V(p,q,s) - p\,V(0,q,,s) - \frac{\partial V(0,q,s)}{\partial x},$$

$$L_xL_yL_t[v_{yy}\,(x,y,t)] = q^2V(p,q,s) - q\,V(p,0,,s) - \frac{\partial V(p,0,s)}{\partial y},$$

$$L_xL_yL_t[v_{tt}\,(x,y,t)] = s^2V(p,q,s) - s\,V(p,q,,0) - \frac{\partial V(p,q,0)}{\partial t}.$$

The inverse double and triple Laplace transform are defined in [23, 24] as follows

$$L_p^{-1}L_s^{-1}[v(p,s)] = v(x,t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} e^{px}dp\,\frac{1}{2\pi i}\int_{d-i\infty}^{d+i\infty} e^{st}ds,$$

$$L_p^{-1}L_q^{-1}L_s^{-1}[v(p,s)] = v(x,t) = \frac{1}{2\pi i}\int_{e-i\infty}^{e+i\infty} e^{px}dp\,\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} e^{qy}dq\,\frac{1}{2\pi i}\int_{d-i\infty}^{d+i\infty} e^{st}ds.$$

2. 1 Theorem ([25]): Let $\alpha,\beta,\gamma > 0$, $n-1 < \alpha \le n, m-1 < \beta \le m$, $r-1 < \gamma \le r$, and $n,m,p \in N$, so that $f \in C^1(R^+ \times R^+ \times R^+)$, $l = max\{n,m,p\}, f^{(1)} \in L_1[(0,a)\times(0,b)\times(0,c)]$ for any $a,b,c > 0$, $|v\,(x,y,t)| \le we^{x\tau 1+y\tau 2+t\tau 3}$, $x > a > 0$, $y > b > 0$, and $t > c > 0$.

Then the double ($D_t^\alpha[v(x,\ t)]$ and $D_x^\zeta[v(x,t)]$) and the triple ($D_t^\alpha[v(x,y,t)]$, $D_y^\beta[v(x,y,t)]$ and $D_x^\zeta[v(x,y,t)]$) Laplace transform of Caputo's fractional derivatives are given by

$$L_xL_tD_t^\alpha[v(x,\ t)] = s^\alpha V(p,s) - \sum_{i=0}^{n-1} s^{\alpha-1-i}L_x\,D_t^i[v(x,0)], \qquad n-1<\alpha<n \tag{1}$$

$$L_xL_tD_x^\zeta[v(x,t)] = p^\zeta V(p,s) - \sum_{k=0}^{r-1} p^{\zeta-1-r}L_t\,D_x^\zeta[v(0,t)], \qquad r-1<\zeta<r \tag{2}$$

$$L_xL_yL_tD_t^\alpha[v(x,y,t)] = s^\alpha V(p,q,s) - \sum_{i=0}^{n-1} s^{\alpha-1-i}L_xL_y\,D_t^i[v(x,y,0)], n-1<\alpha<n \tag{3}$$

$$L_xL_yL_tD_y^\beta[v(x,y,t)] = q^\beta V(p,q,s) - \sum_{j=0}^{m-1} q^{\beta-1-j}L_xL_t\,D_y^j[v(x,0,t)], m-1<\beta<m \tag{4}$$

$$L_xL_yL_tD_x^\zeta[v(x,y,t)] = p^\zeta V(p,q,s) - \sum_{k=0}^{r-1} p^{\zeta-1-r}L_yL_t\,D_x^k[v(0,y,t)], \quad r-1<\zeta<r \tag{5}$$

2. 3 Definition The definition of Mittag-Leffler function is

$$E_\propto(k) = \sum_{i=0}^{\infty}\frac{k^i}{\Gamma(\alpha i+1)}, \qquad k \in C, Re(\alpha) > 0,$$

3 One dimensional fractional diffusion equation

In this section, double Laplace decomposition method has been used to obtain an approximated analytical solution of one dimensional fractional diffusion equation of order α with appropriate initial conditions.

3. 1 The proposed scheme

Consider the fractional diffusion equation in one dimension as

$$\frac{\partial^{\alpha} v(x,t)}{\partial t^{\alpha}} = \frac{\partial^{2} v(x,t)}{\partial x^{2}} + \frac{\partial}{\partial x}\big(F(x)v(x,t)\big), \quad 0 < \alpha \leq 1, \quad t > 0, \quad x > 0 \tag{6}$$

With the initial condition

$$v(x,0) = f(x) \tag{7}$$

where $\frac{\partial^{\alpha}}{\partial t^{\alpha}}(.)$ is the Caputo derivative of order α, $v(x,t)$ represents the probability density function of finding a particle at x in the time t.

In order to obtain the solution of Eq. (6), the modified double Laplace decomposition method is used as follows:

The double Laplace transform for Eq. (6) is

$$s^{\alpha} L_x L_t[v(x,t)] - s^{\alpha-1} V(p,0) = L_x L_t\left(\frac{\partial^{2} v}{\partial x^{2}} + \frac{\partial}{\partial x}(F(x)v(x,t)\right) \tag{8}$$

Applying the differentiation property of the Laplace transform, we get

$$L_x L_t[v(x,t)] = \frac{f(x)}{s} + \frac{1}{s^{\alpha}} L_x L_t\left(\frac{\partial^{2} v(x,t)}{\partial x^{2}} + \frac{\partial}{\partial x}(F(x)v(x,t)\right) \tag{9}$$

By implementing of double inverse Laplace transformation to Eq. (9) we obtain

$$v(x,t) = L_p^{-1} L_s^{-1}\left(\frac{f(x)}{s}\right) + L_p^{-1} L_s^{-1}\left(\frac{1}{s^{\alpha}} L_x L_t\left(\frac{\partial^{2} v}{\partial x^{2}} + \frac{\partial}{\partial x}(F(x)v(x,t)\right)\right) \tag{10}$$

The infinite series solution of Laplace Adomian decomposition function $v(x,t)$ is

$$v(x,t) = \sum_{n=0}^{\infty} v_n(x,t) \tag{11}$$

Substituting by Eq. (11) into Eq. (10), we get

$$\sum_{n=0}^{\infty} v_n(x,t) = L_p^{-1}L_s^{-1}\left(\frac{f(x)}{s}\right) + L_p^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}L_xL_t\left(\frac{\partial^2 v_n}{\partial x^2} + \frac{\partial}{\partial x}(F(x)v_n(x,t))\right)\right) \quad (12)$$

Using Laplace Adomian decomposition method, we get:

$$v_0(x,t) = L_p^{-1}L_s^{-1}\left(\frac{f(x)}{s}\right) \quad (13)$$

and the rest of components v_{n+1}, for $n \geq 0$, are given by

$$v_{n+1}(x,t) = L_p^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}L_xL_t\left(\frac{\partial^2 v_n(x,t)}{\partial x^2} + \frac{\partial}{\partial x}(F(x)v_n(x,t))\right)\right), \quad n \geq 0 \quad (14)$$

3. 2 Illustrative examples

In this section, three examples are given to illustrate the applicability of the presented method and all of them are performed on the computer by using the Mathematica 9 program.

Example 1. Taking $F(x) = x$ in Eq. (6) and Substituting with $f(x) = 1$ in Eq. (7) we get

$$\frac{\partial^\alpha v(x,t)}{\partial t^\alpha} = \frac{\partial^2 v(x,t)}{\partial x^2} + \frac{\partial}{\partial x}\big(xv(x,t)\big), \quad 0 < \alpha \leq 1, \quad t > 0, \quad x > 0, \quad (15)$$

$$v_0(x,0) = 1. \quad (16)$$

As reported by the above steps in section 3.1, we have

$$v_{n+1}(x,t) = L_p^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}L_xL_t\left(\frac{\partial^2 v_n(x,t)}{\partial x^2} + \frac{\partial}{\partial x}(xv_n(x,t))\right)\right) \quad (17)$$

The zeroth component v_0 should contain initial condition and the source term, so we choose $v_0 = 1$ and the other components $v_{n+1}, n \geq 0$ is given by the relation (17), by setting $n = 0$ in Eq. (17), we have

$$v_1(x,t) = L_p^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}L_xL_t\left(\frac{\partial^2 v_0(x,t)}{\partial x^2} + \frac{\partial}{\partial x}(xv_0(x,t))\right)\right), \quad (18)$$

$$v_1(x,t) = \frac{t^\alpha}{\Gamma[1+\alpha]}, \quad (19)$$

Similarly, when $n = 1$, we get

$$v_2(x,t) = L_p^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}L_xL_t\left(\frac{\partial^2 v_1(x,t)}{\partial x^2} + \frac{\partial}{\partial x}(xv_1(x,t))\right)\right), \quad (20)$$

$$v_2(x,t) = \frac{t^{2\alpha}}{\Gamma[1+2\alpha]}, \tag{21}$$

Hence, the solution of Eq. (15) is given by

$$v(x,t) = \sum_{n=0}^{\infty} v_n(x,t) = v_0 + v_1 + v_2 + v_{3+\cdots} \tag{22}$$

$$v(x,t) = \sum_{k=0}^{\infty} \frac{(t^{\alpha})^k}{\Gamma[1+\alpha\, k]} = E_{\alpha}(t^{\alpha}) \tag{23}$$

where $E_{\alpha}(\mathrm{t}) = \sum_{k=0}^{\infty} \frac{t^k}{\Gamma[1+\alpha\, k]}$, $\alpha > 0$ is the Mittag-Leffler function in one parameter.

We get the same results as in [26]. In the above example, Eq. (23) has been used to draw the three dimensional figure for the approximate solution $v(x,t)$ with respect to x and t for $\alpha = 1$, 0.9, 0.8 and 0.7 as shown in figure 1. Figure 2 evinces the comparison of the absolute error between the exact and approximate solution for example 1 for $\alpha = 1$ at $t = 0.5$. it is clear that the approximate solution has good agreement with the given exact solution.

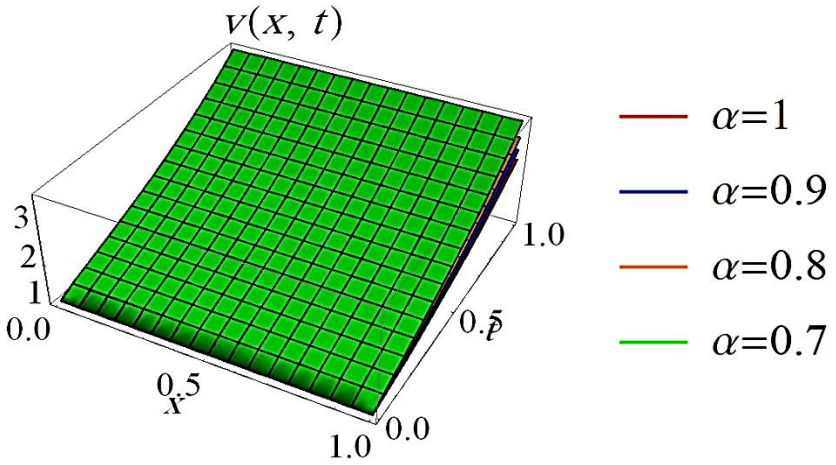

Fig.1. Plot of the field variable $v(x,t)$ versus *x* and *t* for different values of α.

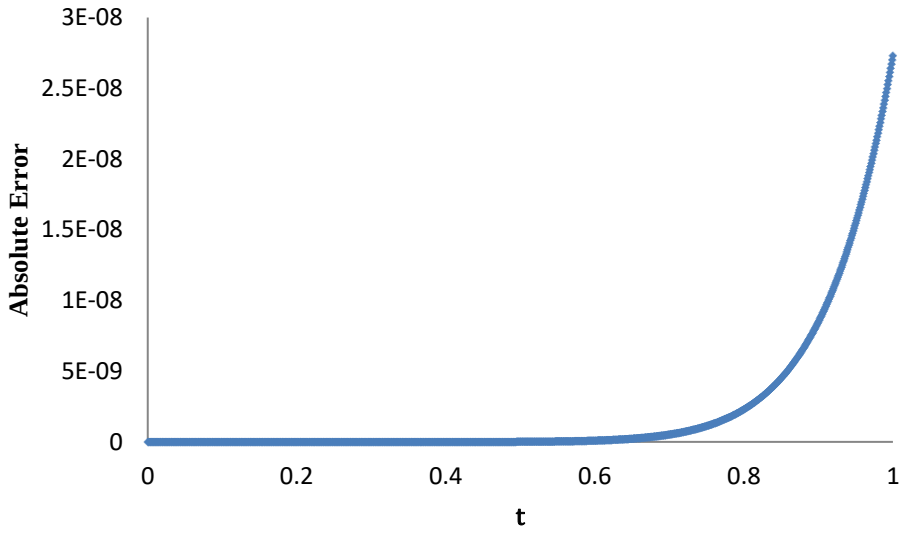

Fig.2. Absolute error graph for numerical and exact solutions versus t for $\alpha = 1$ and $t = 0.5$.

Example 2. Taking $F(x) = x$ in Eq. (6) and Substituting with $f(x) = x$ in Eq. (7) we get

$$\frac{\partial^\alpha v}{\partial t^\alpha} = \frac{\partial^2 v}{\partial x^2} + \frac{\partial}{\partial x}\big(xv(x,t)\big), \quad 0 < \alpha \leq 1, \quad t > 0, \quad x > 0, \tag{24}$$

$$v_0(x,0) = x. \tag{25}$$

As stated by the above steps in section 3.1, we have

$$v_{n+1}(x,t) = L_p^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}L_xL_t\left(\frac{\partial^2 v_n}{\partial x^2} + \frac{\partial}{\partial x}(xv_n(x,t)\right)\right) \tag{26}$$

We set $v_0 = x$ which contains the initial condition and the other components $v_{n+1}, n \geq 0$ are given by the relation (15), setting $n = 0$ in Eq. (15), we get

$$v_1(x,t) = L_p^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}L_xL_t\left(\frac{\partial^2 v_0}{\partial x^2} + \frac{\partial}{\partial x}(xv_0(x,t)\right)\right), \tag{27}$$

$$v_1(x,t) = \frac{2\,xt^\alpha}{\Gamma[1+\alpha]}. \tag{28}$$

Similarly, when $n = 1$, we obtain

$$v_2(x,t) = L_p^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}L_xL_t\left(\frac{\partial^2 v_1}{\partial x^2} + \frac{\partial}{\partial x}(xv_1(x,t)\right)\right), \tag{29}$$

$$v_2(x,t) = \frac{4\,x\,t^{2\alpha}}{\Gamma[1+2\alpha]}. \tag{30}$$

Hence, the solution of Eq. (24) is given by

$$v(x,t) = \sum_{n=0}^{\infty} v_n(x,t) = v_0 + v_1 + v_2 + v_{3+\ldots} \tag{31}$$

$$v(x,t) = \sum_{k=0}^{\infty} \frac{(2t^{\alpha})^k}{\Gamma[1+\alpha\, k]} = x\, E_{\alpha}(t^{\alpha}) \tag{32}$$

The same solution has been obtained by Cetinkaya [26]. Also this solution is in complete agreement with [13, 14] for $\alpha = 1/2$. Figure 3 shows that the approximate solution of example 2 for $\alpha = 1$, 0.9, 0.8 and 0.7. To show the accuracy of the proposed method solution, the absolute error for $\alpha = 1$ at $t = 0.5$ is given in figure 4.

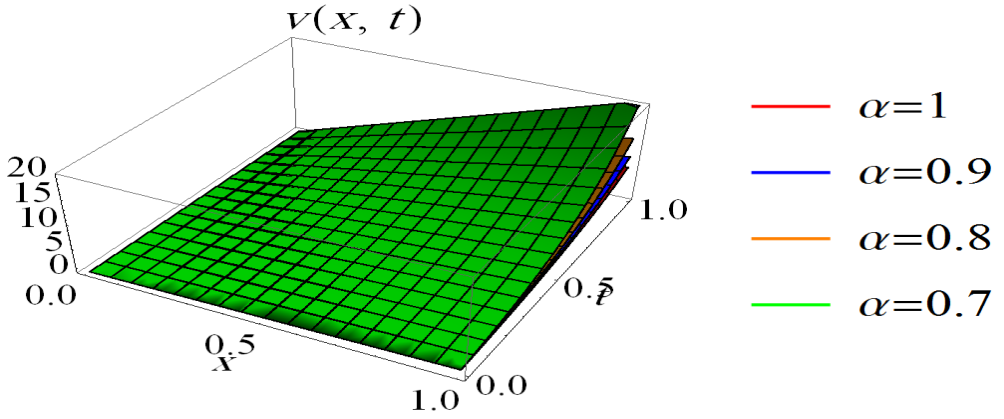

Fig.3. Plot of the field variable $v(x,t)$ versus x and t for different values of α.

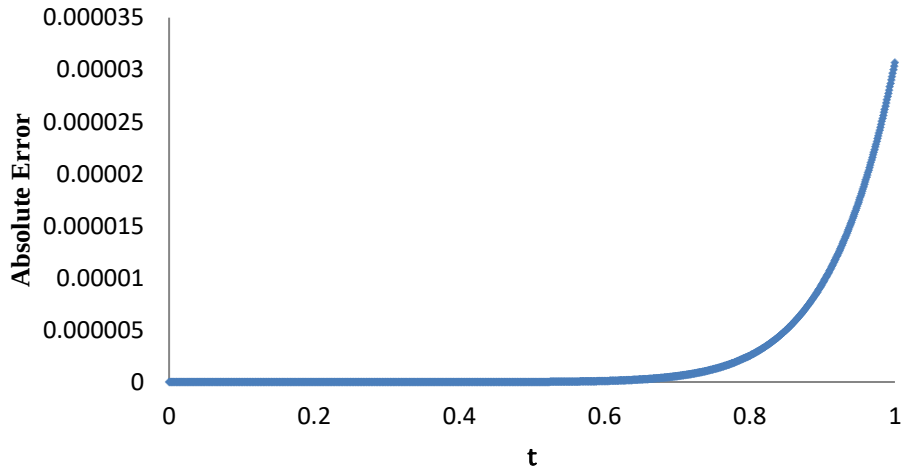

Fig.4. Absolute error graph for numerical and exact solutions versus t for $\alpha = 1$ and $t = 0.5$.

Example 3. Taking $F(x) = e^{-x}$ in Eq. (6) and Substituting with $f(x) = e^{x}$ in Eq. (7) we get

$$\frac{\partial^{\alpha} v(x,t)}{\partial t^{\alpha}} = \frac{\partial^{2} v(x,t)}{\partial x^{2}} + \frac{\partial}{\partial x}\left(e^{-x} v(x,t)\right), \quad 0 < \alpha \leq 1, \quad t > 0, \quad x > 0, \tag{33}$$

$$v_0(x,0) = e^{x}.$$

In the same manner, we obtain that

$$v(x,t) = e^{x} E_{\alpha}(t^{\alpha}) \tag{34}$$

Our results are the same obtained in [26]. The approximate solution of the above example for various values of α and the absolute error when $\alpha = 1$ at $t = 0.5$ are shown in figs.5 and 6, receptively.

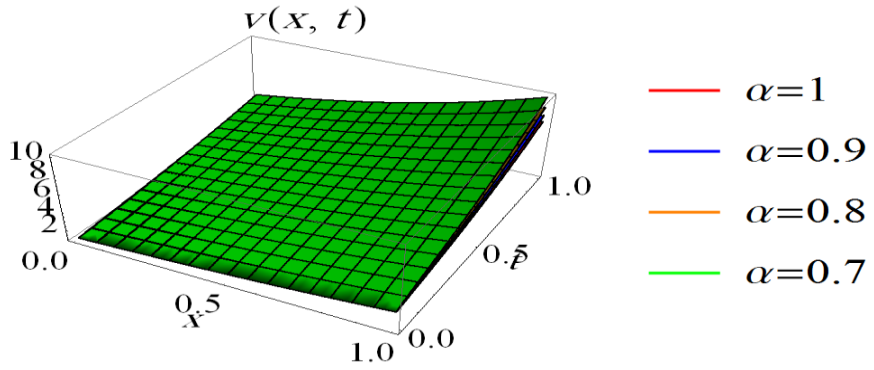

Fig.5. Plots of the field variable $v(x,t)$ versus *x* and *t* for different values of α.

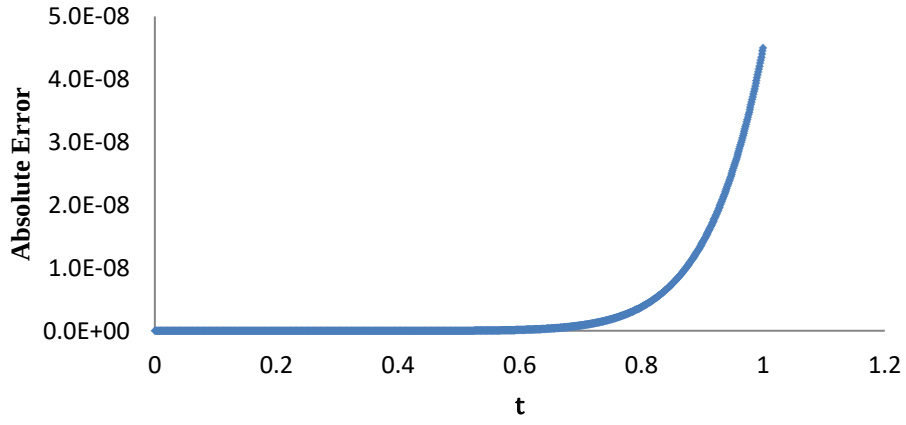

Fig.6. Absolute error graph for numerical and exact solutions versus t for $\alpha = 1$ and $t = 0.5$.

4 Two dimensional fractional diffusion equation

In this section, the triple Laplace Adomain decomposition method is applied to fractional two-dimensional space-time diffusion equation equation.

4. 1 The proposed scheme

Consider a time-fractional two-dimensional PDE as

$$\frac{\partial^{\alpha} v(x,y,t)}{\partial t^{\alpha}} + \frac{\partial^{3} v(x,y,t)}{\partial x^{3}} + \frac{\partial^{3} v(x,y,t)}{\partial y^{3}} = 0, \qquad 0 < \alpha \leq 1, \;\; y \geq 0, \;\; x \leq 1 \qquad (35)$$

With the given initial condition

$$v(x,y,0) = v_0(x,y) \qquad (36)$$

The exact solution for this fractional PDE is $v(x,y,t) = cos(y + 2t + x)$ for $\alpha = 1$. Hence; at $t = 0$, the initial condition can be written by $v_0(x,y) = cos(y + x)$. This equation represents two-dimensional diffusion in porous media. To find the solution of Eq. (35), we apply triple Laplace Adomian decomposition method on Eq. (35) as follows:

Taking the triple Laplace transform for Eq. (35), we obtain

$$s^{\alpha} L_x L_y L_t[v(x,y,t)] - s^{\alpha-1} V(x,y,0) = -L_x L_y L_t\left(\frac{\partial^{3} v(x,y,t)}{\partial x^{3}} + \frac{\partial^{3} v(x,y,t)}{\partial y^{3}}\right) \qquad (37)$$

Applying the differentiation property of the Laplace transform, we have

$$L_x L_y L_t[v(x,y,t)] = \frac{v_0(x,y)}{s} - \frac{1}{s^{\alpha}} L_x L_y L_t\left(\frac{\partial^{3} v(x,y,t)}{\partial x^{3}} + \frac{\partial^{3} v(x,y,t)}{\partial y^{3}}\right) \qquad (38)$$

The triple inverse Laplace transform is implemented on Eq. (38) to obtain

$$v(x,y,t) = L_p^{-1} L_q^{-1} L_s^{-1}\left(\frac{v_0(x,y)}{s}\right) + L_p^{-1} L_q^{-1} L_s^{-1}\left(\frac{1}{s^{\alpha}} L_x L_y L_t\left(\frac{\partial^{3} v(x,y,t)}{\partial x^{3}} + \frac{\partial^{3} v(x,y,t)}{\partial y^{3}}\right)\right) \qquad (39)$$

The infinite series solution of Laplace Adomian decomposition function $v(x,t)$ is

$$v(x,y,t) = \sum_{n=0}^{\infty} v_n(x,y,t), \qquad (40)$$

By substituting Eq. (40) into Eq. (39), we get

$$\sum_{n=0}^{\infty} v_n(x,y,t) = L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{v_0(x,y)}{s}\right)$$

$$= +L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}L_xL_yL_t\left(\frac{\partial^3 v_n(x,y,t)}{\partial x^3}+\frac{\partial^3 v_n(x,y,t)}{\partial y^3}\right)\right) \quad (41)$$

Using Laplace Adomian decomposition method, we get:

$$v_0(x,y,t) = L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{F_1(p,q)}{s}\right), \quad (42)$$

and the rest of components v_{n+1}, for $n \geq 0$, are given by

$$v_{n+1}(x,y,t) = L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}L_xL_yL_t\left(\frac{\partial^3 v_n(x,y,t)}{\partial x^3}+\frac{\partial^3 v_n(x,y,t)}{\partial y^3}\right)\right) \quad (43)$$

4. 2 Illustrative example

Example 4. Taking $v_0(x,y,0) = cos(y + x)$ which contains initial condition and the source term**.**

By applying the above steps, we obtain when $n = 0$ in Eq. (43),

$$v_1(x,y,t) = L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}L_xL_xL_t\left(\frac{\partial^3 v_0(x,y,t)}{\partial x^3}+\frac{\partial^3 v_0(x,y,t)}{\partial y^3}\right)\right), \quad (44)$$

$$v_1(x,y,t) = -\frac{e^{-iy}t^\alpha\left(iCos[x]-ie^{2iy}Cos[x]+Sin[x]+e^{2iy}Sin[x]\right)}{\Gamma[1+\alpha]}. \quad (45)$$

Similarly, when $n = 1$, we can determine

$$v_2(x,y,t) = L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}L_xL_xL_t\left(\frac{\partial^3 v_1(x,y,t)}{\partial x^3}+\frac{\partial^3 v_1(x,y,t)}{\partial y^3}\right)\right), \quad (46)$$

$$v_2(x,y,t) = -\frac{2e^{-ix-iy}(1+e^{2ix+2iy})t^{2\alpha}}{\Gamma[1+2\alpha]}. \quad (47)$$

Hence, the solution of Eq. (35) is given by

$$v(x,y,t) = \sum_{n=0}^{\infty} v_n(x,y,t) = v_0 + v_1 + v_2 + v_{3+\cdots} \quad (48)$$

Hence, we get the solution in series form as

$$v(x,y,t) = cos(y + x) + \frac{-2e^{-ix-iy}(1+e^{2ix+2iy})t^{2\alpha}}{\Gamma[1+2\alpha]} + \frac{-4ie^{-ix-iy}(-1+e^{2ix+2iy})t^{3\alpha}}{\Gamma[1+3\alpha]} + \cdots \quad (49)$$

The approximate series solution in Eq. (49) converges rapidly to exact solution after few approximate terms. For $\alpha = 1$, above series solution can be written in closed form as

$$v(x,y,t) = cos(2t)cos(y + x) - sin(2t)sin(y + x) = cos(y + x + 2t) \quad (50)$$

Our results are the same obtained in [16]. According to the above equations, in figure 7 the approximated solution has been plotted for $\alpha = 1$, 0.9, 0.8 and 0.7. Figures 8 and 9 show the absolute error for $\alpha = 1$ at $t = 0.5$ in three dimensional and two dimensional when = 0.5 , respectively.

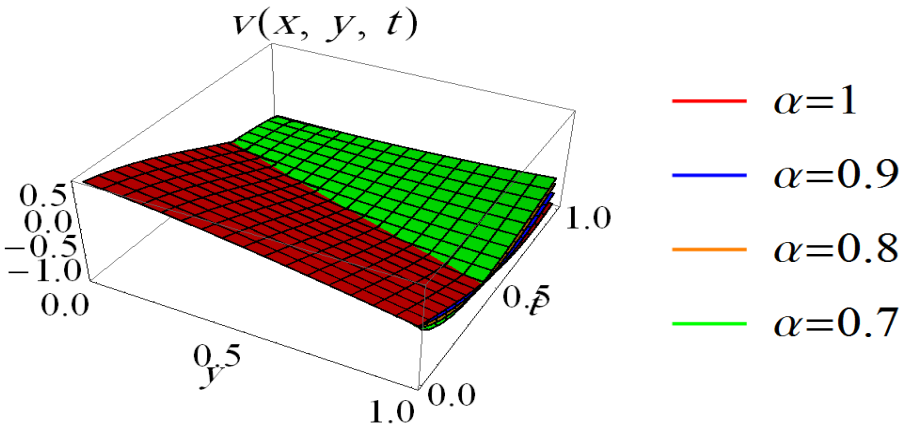

Fig.7. Plots of the field variable $v(x, y, t)$ versus y and t at $x = 0.5$ for different values of α.

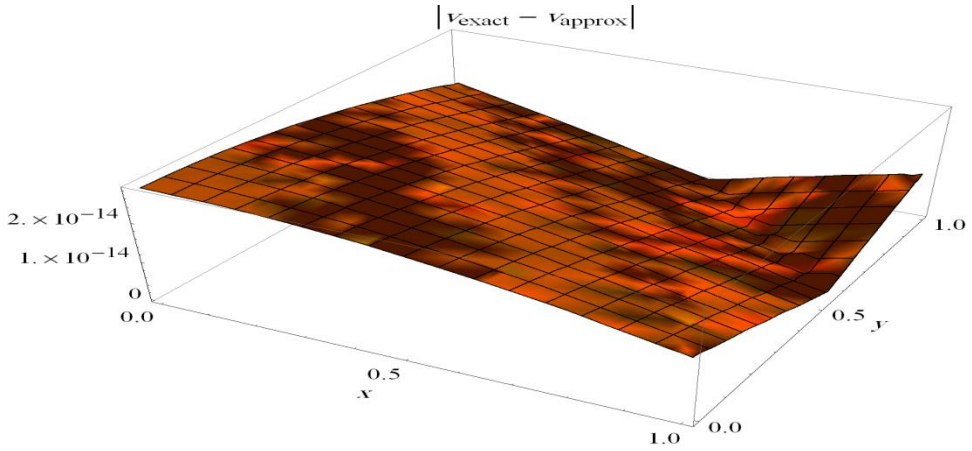

Fig.8. Absolute error graph for numerical and exact solutions versus *x* and *y* for α =1 and *t* = 0.5.

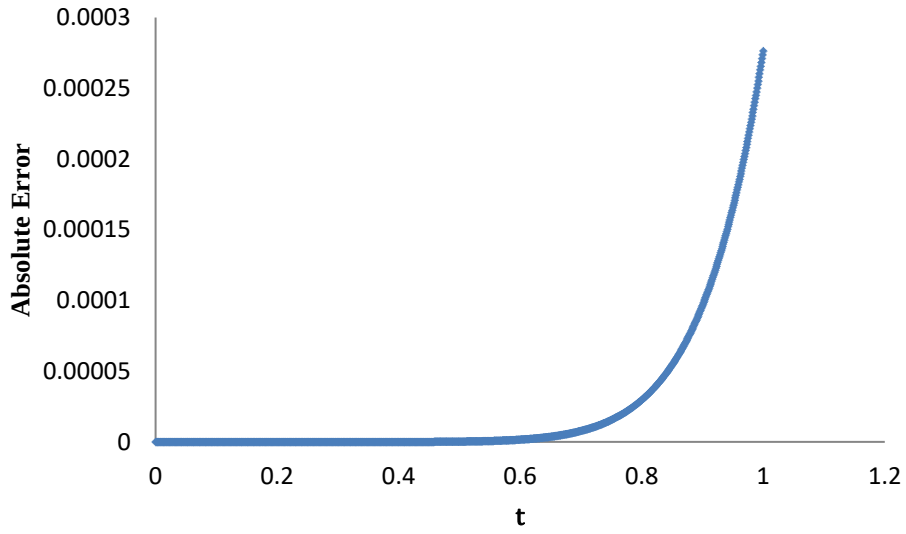

Fig.9. Absolute error graph for numerical and exact solutions versus t for α =1 and x, y = 0.5.

Conclusion

In this paper, we exhibit the applicability of double and triple Transform with the Adomain decomposition method to solve one and two-dimensional fractional diffusion equations respectively with different initial conditions using Caputo's fractional derivative. The procedure of calculation shows that the presented method converges rapidly to the exact solution and can be counted as the competitive method. The illustrative examples demonstrate the reliability and efficiency of the present technique. This method can be applied to other fractional order partial differential equations.

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