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Determining Optimal Locations for Emergency Service Centers with Limited Capacity

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Abstract

The rational management of emergency services centers focuses on two important goals, the optimal positioning of these centers to be able to provide services in the fastest possible way, and the second is to ensure that these centers can provide, with no delay, service to every one upon his/her request, especially in emergency cases when demand on these services are at the peak which means that in urban design it is very important to locate the emergency service centers in a position that satisfy both the two above goals. In fact, several research works concerned with determining the optimum location of the service centers but with a little concern of the limited capacity of each center. This research discusses how to achieve these two goals by treating the problem mathematically. Limitation of the capacity of each center is also a matter of concern. Finally, an effective computer algorithm is provided to illustrate the practical implication of the performed mathematical solution.

Keywords: Emergency Services, optimization, location, capacity

1. Introduction:

The sudden and brutal outbreak of COVID-19 pandemic has thrown the governments all over the world into a frenzy tough situation and has put them onto the frontlines in the response to the crises forcing them to deal and respond quickly to the crises without any prepared strategy or roadmap. Public servants especially those who are working in public health service centers, have to make plans very quickly to face the huge losses caused by the epidemic in the delivery of services, especially in the field of health care. [1]

Since public health centers are essential for providing health care to citizens, they become very sensitive to external factors such as the outbreak of pandemics. This outbreak put several public health services under pressure and to encounter unexpected additional demand of health care. [2]

The issue of optimal positioning of emergency (health service) facility centers has been and continues to be of interest to many researchers, both from a mathematics and management science perspective. Several mathematical researchers have presented different formulations, as well as suggesting solutions to this problem [3-8]. The main theme of these researches has been based on the mathematical study of graph theory [9-12]. Accordingly, researchers in different fields have also provided many real life applications for this problems, for examples see [5,6,8,13,14]

From this perspective, Zimmermann [15] formulated and solved one of the basic versions of this problem which is regarding several callers for the service and several service center are connected in one service system emphasizing on minimizing the worst case of the spatial separation between the caller for the service and the service center that will provide the optimal service to this caller , This treatment may be considered the base of many forms and cases that are branching from this problem to cover and investigate different cases and their complications [16-23].

In all of these works the problems were concerning optimizing the location(s) of the center(s) of emergency facilities considering different situations of the proposed positions to build the centers, and possible cases of traffic network which connects the centers and the requesters for the service and also the nature of the inputs of the problem or even the different priorities of the requesters.

Nevertheless, in almost all these researches and treatments, the capacity of each facility center has not taken into account, as if it was assumed that any emergency service center could serve any number of service seekers simultaneously, which is unrealistic and impractical as well.

In the present work the authors reformulate the Zimmermann problem [15] by adding the capacity of each service center as a parameter to measure the number of customers that center can serve at one time and investigate the problem mathematically, then reconstruct the modified algorithm that determines the optimal locations for the service centers and see if the capacity of the centers is the proposed service is sufficient to provide the service for all expected calls or not.

The paper is organized in the following way: Section 2 introduces the problem statement; Section 3 includes mathematical derivations and treatments. Section 4 performs the suggested algorithm that yielding the solution and the explanatory example to explain how is the algorithm working. Section 5 discusses a summary of the present work and recommends some items for extending this approach in our future work.

2. The Research Problem:

In the process of assigning the locations of emergency services centers, it is possible to choose suitable places for these centers so that they are close to all potential service seekers, but if the capacity of each of these centers is not taken into account, a sudden and simultaneous heavy demand may occur on one or more of them, making it unable to provide the service to all its requesters, which causes major problems, especially that we are in the process of emergency services that require a quick and completed service, and these cases have already appeared in many health systems at the beginning of the Covid-19 pandemic. So here we are going to reformulate and one of the versions for the problems of determining the optimum emergency services centers formulated by Zimmermann [15] taking into account the capacity of each service center.

3. Defining the Problem:

Suppose that the service centers of some emergency services can be located in an area such that each one must be located on one street (line segment $\overline{A_j B_j},\ j \in \{1,2,...n\}$) and the length of each of these streets ($A_j B_j = d_j$) is known.

Each of these centers may serve any of m service requester that are located in certain places (represented by points) "$Y_i, i \in \{1, 2, ..., m\}$" supposing that lengths of the routes connecting any requester "Y_i" and the end points of the street "$\overline{A_j B_j}$" are known and denoted by "a_{ij}, b_{ij}" respectively.

Let us consider that paths connecting each center "S_j" to any requester must passes via one of the end points of "$\overline{A_j B_j}$" which results two possible paths that are connecting any service center and service requester (Figure 1).

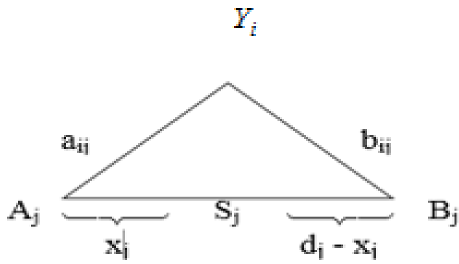

Figure 1. Paths connecting each service center to any requester

To respond to any help request sent by caller "Y_i", the service management should send the required personnel from the nearest center, and take the smallest route to reach the caller in the fastest way. So, the distance to be traveled to the requester position is the minimum of the following:

- "$x_j + a_{ij}$"
- "$(d_j - x_j) + b_{ij}$"

So, the smallest distance connecting "S_j" and "Y_i" can be defined as: "min { $x_j + a_{ij}$, $(d_j - x_j) + b_{ij}$ }".

3.1 Arranging the Callers

In the problem we are discussing, we will also assume that there is a hierarchy of the distances that separate service seekers and any service center as follows:

Assumption 1: (see [20])

"$\forall j \in \{1, 2, \ldots, n\}$, *a* permutation *π = (1*, 2*, . . . , m*)* of the indices *(1, 2, . . . ,m)* can be found such that:

$$(a_{i_1^* j}, b_{i_1^* j}) \le (a_{i_2^* j}, b_{i_2^* j}) \le ,\ldots\ldots, \le (a_{i_m^* j}, b_{i_m^* j})\text{"}$$

Therefore, for every "$j \in \{1, 2, \ldots, n\}$" the set "$\{Y_i,\ i \in \{1, 2 \ldots m\}\}$" can be arranged with respect to the distances connecting any of them with the extreme points of the line segment "$\overline{A_j B_j}$".

This assumption was proposed by Zimmermann in [21] to overcome the computational complexity of the problem that appeared after Hudec proved in [4] that the solution of the problem cannot be reached by deterministic polynomial time algorithm (NP-hard problem).

3.2 Limited Capacity

As mentioned above the problem in its original form (see [1] and [17]) has ignored the capacity of each service center (which represents the maximum number of callers it can serve simultaneously), which means that each service center is supposed to serve any number of callers at one time. This assumption is not Realistic and can cause big problems if a large number of callers need help at the same time.

Here we are going to assume that each service center "S_j" has given capacity "$C_j \in \mathbb{Z}^+$" referring to the maximum number of callers can be served by this center in the same time.

To avoid the situation in which the service center can be set as a server for a number of callers exceeding its capacity, we will define the function of the shortest route connecting the service center "S_j" with the requester "Y_i" as:

$$\text{“}\ r_{ij}(x_j) = \begin{cases} min\ \{x_j + a_{ij}, (d_j - x_j) + b_{ij}\} & \text{if } i \le C_j \\ \infty & \text{if } i > C_j \end{cases}\ \text{“} \qquad (1)$$

Such that "i" represents the order of the caller according the above ordering assumption.

It is not difficult to see that imposing the previous ordering assumption will satisfy the following result:

$$\text{, “}\ \forall x \in [0, d_j],\ r_{1j}(x) \le r_{2j}(x) \le \ldots \le r_{mj}(x)\ \text{”}$$

It is natural that any request for service will be responded to from the nearest emergency center, then we can express the distance function necessary to respond to the requester Y_i by:

$$\text{“}\ f_i(x) = \min_{j \in \{1,2,\ldots n\}} \{r_{ij}(x_j)\},\quad \text{where } x = (x_1, x_2, \ldots, x_m) \qquad (2)\text{”}$$

Because the problem we are discussing is related to emergency services, we must consider a possible worst-case scenario, which is the furthest distance between any service caller and the emergency center that will assist, that can be describes as:

$$\text{“}\ F(x) = \max_{i \in \{1,2,..,m\}} (f_i(x)) \qquad (3)\text{”}$$

The aim of the problem is to minimizes the function “ $F(x)$ ” by finding the optimum tuple

“ $x=(x_1,x_2,\ldots,x_n)$ ” such that “ $F(x)$ ” is less than given positive number “ α ”.

The problem can be expressed as:

Definition 1 (Problem P_1):

“Minimize F (x)

S.T $F(x) \le \alpha$,

$0 \le x_j \le d_j$ ”

3.3 An Equivalent Formulation:

Definition 2:

For any positive number “ α ” and for any “ $i \in \{1,2,\ldots,m\}$ and $j \in \{1,2,\ldots,n\}$ ” define the feasible solution set “ $V_{ij}(\alpha)$ ” as:

$$“V_{ij}(\alpha)=\begin{cases}\{x : x\in[0,d_j]\wedge r_{ij}(x)\le\alpha\} & \text{if } i\le C_j \\ \phi & \text{if } i>C_j\end{cases}”$$

And

$$“\ M\ (\alpha)\ =\ \left\{x=\left(x_1,\ x_2,\ldots x_n\right)\ :\ 0\ \le\ x_j\ \le\ d_j,\ F\left(x\right)\ \le\ \alpha\right\}”$$

The following assumption can be obtained from the above assumption:

Assumption 2

For each “ $j\in\left\{1,\ 2,\ \ldots,\ n\right\}$ ”, there exists a permutation “ $\pi\ =\ \left(1^*,\ 2^*,\ \ldots,m^*\right)$ ” of the indices

“ $\left(1,\ 2,\ \ldots,\ m\right)$ ”

Such that:

For each positive scalar “ α ”, “ $V_{i_1^* j}(\alpha)\supseteq V_{i_2^* j}(\alpha)\supseteq\ldots\supseteq V_{i_m^* j}(\alpha)$ ”.

So, problem ($P1$) can be reformulated as:

Definition 3 (problem $P2$)

$$\text{Minimize: } \alpha$$

$$\begin{aligned} \text{Subject to} \quad & \alpha > 0 \\ & M(\alpha) \neq \phi, \\ & 0 \le x_j \le d_j \text{"} \end{aligned}$$

Note: For a full mathematical discussion of this problem, see [3] and [17]**.**

4. The Algorithm

We are ready to present our modified version of the original algorithm mentioned in [17] to solve the problem (P2) that can be used to determine the optimal possible locations of emergency service centers or to demonstrate that there is no workable solution to the problem due to the limited capacity of each service center.

The steps of the algorithm can be written as follows:

1) Read:

- *"m"* (The number of callers)
- *"n"* (The number of Service Centers)
- "α*" (Maximum threshold) (optional)

2) "$\forall i \in \{1, 2, \ldots, m\}, \forall j \in \{1, 2, \ldots, n\}$." Input the values:

- "C_j" (The maximum capacity of each center).
- "a_{ij}, b_{ij}, d_j" (The Spatial Parameters).

3) Verify the ordering assumption;

If it is hold go to Step 4.

Else, write "the assumption is not hold" and stop.

4) For every caller "i" and street "j", Calculate the minimum value "δ_{ij}" as:

"If $i \le C_j \to \delta_{ij} = min\left\{a_{ij}, b_{ij}\right\}$"
Else $\delta_{ij} \to = \infty$

5) For every caller "i" and street "j", Calculate the maximum value as "ρ_{ij}":

"If $i \le C_j \to \rho_{ij} = ½\left(b_{ij} + a_{ij} + d_j\right)$"
Else $\to \rho_{ij} = \infty$

6) "$\forall i \in \left\{1, 2, \ldots, m\right\}$.", put: "$\alpha_i = min\left\{\delta_{ij} : j = 1, \ldots, n\right\}$".

7) If the* is given, then "$\forall i \in \left\{1, 2, \ldots, m\right\}$." Update "$\alpha_i$" as:

"$\alpha_i = \min\{\alpha^*, \alpha_i\}$."

8) determine the threshold value "α" as "$\alpha = max\ \{\alpha_i : i = 1, \ldots, m\}$"

9) For every caller "i" and street "j", Calculate values:

"$L_{ij}(\alpha) = \alpha - a_{ij}$", "$H_{ij}(\alpha) = d_j + b_{ij} - \alpha$".

10) For every caller "i" and street "j", Determine the set "$V_{ij}\left(\alpha\right)$" according to definition (2) and then write the matrix "$V_{ij}\left(\alpha\right)$".

11) "$\forall i \in \left\{1, 2, \ldots, m\right\}$." Determine an index "$j\left(i\right), j \in \left\{1, 2, \ldots, n\right\}$," such that:

"$V_{ij}\left(\alpha\right) \ne \phi$." (If there is more than one, then break the tie arbitrary).

If "$V_{ij}\left(\alpha\right) = \phi$" for each index "$j \in \left\{1, 2, \ldots, n\right\}$," then: **Go to step 13.**

12) "$\forall j \in \left\{1, \ldots, n\right\}$" put:

"$P_j\left(\alpha\right) = \{i : i \in \left\{1, \ldots, m\right\}, V_{ij}\left(\alpha\right) \ne \varphi\}$"

"$V_j = \bigcap_{i \in P_j(\alpha)} V_{ij}(\alpha)$"

$$“\ x_j^{opt} \in \begin{cases} \mathrm{V_j} & \text{if} \quad \mathrm{P_j}(\alpha) \neq \phi \\ [0,\mathrm{d_j}] & if \quad \mathrm{P_j}(\alpha) = \phi \end{cases} ”$$

Go to step 14.

Remark: step (3) guaranteed that “$\forall j \in \{1, \ . \ . \ . \ , \ n\}$” the sets “$V_{ij}(\alpha)$” are nested which makes the implementation of step (12) possible in polynomial time (for more details see [4] and [22])

13) Write “no solution” and STOP.

14) Write “the solution is: $(x_1^{opt}, x_2^{opt}, \ldots\ldots\ldots, x_n^{opt})$ ” and Stop.

4.1 Example:

Here we will introduce an example to explain the way of work of the above algorithm:

An ambulance service will be provided to four potential callers (i.e. m = 4) through three ambulance centers (i.e. n = 3). Each of these centers will be constructed alone on one street, the streets are equal in length $d_1 = d_2 = d_3 = 5$. The centers have capacities 3, 2 and 1 respectively (i.e., $C_1 = 3$, $C_2 = 2$ and $C_3 = 1$). Also, there are two lanes from each caller to each proposed street. (See figure 2)

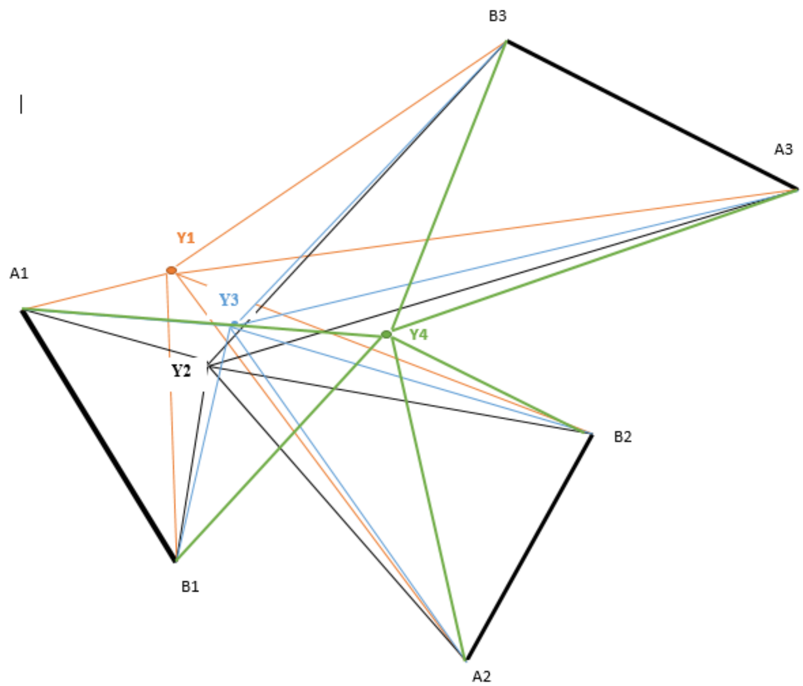

Figure 2. Example to explain the way of work of the above algorithm

Let the lengths of the two routes connecting each caller and the end points of a street are given by:

$$"(a_{ij}, b_{ij}) = \begin{bmatrix} (1,2) & (6,7) & (6,5) \\ (2,2) & (5,5) & (7,7) \\ (3,4) & (4,4) & (8,9) \\ (4,5) & (3,3) & (4,5) \end{bmatrix}".$$

If the goal of the management dictates that, distance connecting a caller and the ambulance station that will serve him is not allowed to exceed 10 units of distance (i.e. $\alpha^* = 10$).

By examining the matrix, we can see that the order of the assumptions is kept.

By executing steps 4, 5, 6, 7 of the algorithm, we obtain:

$$``\delta_{ij} = \begin{bmatrix} 1 & \infty & \infty \\ 2 & \infty & \infty \\ 3 & 4 & \infty \\ \infty & 3 & 4 \end{bmatrix}" \quad ``\rho_{ij} = \begin{bmatrix} 4 & \infty & \infty \\ 4.5 & \infty & \infty \\ 6 & 6.5 & \infty \\ \infty & 5.5 & 7 \end{bmatrix}" \quad ``\tilde{\alpha}_i = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 3 \end{bmatrix}"$$

Step 8 determines the maximum feasible threshold "α = 3".

By steps 9, 10, 11 of the algorithm, we obtain:

$$``L_{ij} = \begin{bmatrix} 2 & -3 & -3 \\ 1 & -2 & -4 \\ 0 & -1 & -5 \\ -1 & 0 & -1 \end{bmatrix}" \qquad ``H_{ij} = \begin{bmatrix} 5 & 9 & 7 \\ 4 & 7 & 9 \\ 6 & 6 & 11 \\ 7 & 5 & 7 \end{bmatrix}"$$

And

$$``V_{ij}(\alpha) = \begin{bmatrix} [0,2]\bigcup\{5\} & \phi & \phi \\ [0,1]\bigcup[4,5] & \phi & \phi \\ \{0\} & \phi & \phi \\ \phi & \{0\}\bigcup\{5\} & \phi \end{bmatrix}"$$

Steps 12, 13, 14 of the algorithm, give the values of:

$$``P_j = \left[\{1,2,3\} \quad \{4\} \quad \phi\right]"$$

So we can see that the center number one cans serve the callers 1, 2 and 3 and center number two will serve caller number 4 and there is no need to build center number 3.

And "$V_j = [\{0\} \quad \{0\} \cup \{5\} \quad \phi]$"

Therefore "$x^{opt} = \begin{bmatrix} x_1^{opt} \\ x_2^{opt} \\ x_3^{opt} \end{bmatrix} = \begin{bmatrix} \{0\} \\ \{0\} \\ \phi \end{bmatrix}$ or $x^{opt} = \begin{bmatrix} x_1^{opt} \\ x_2^{opt} \\ x_3^{opt} \end{bmatrix} = \begin{bmatrix} \{0\} \\ \{5\} \\ \phi \end{bmatrix}$"

We can conclude the following: (see figure 3)

i. Center number 1 has to be located at the first end-point of the first street that is to serve each of caller number 1, 2 and 3.

ii. Center number 2 can be located at any of the end-points of street number 2 that is to serve caller number 4.

iii. The longest distance to travelled from a caller and its associated center is 3 distance units which smaller than the dictated goal of the management (5 units of distance).

iv. No need to build the service center number 3 because it will not improve the quality of the service.

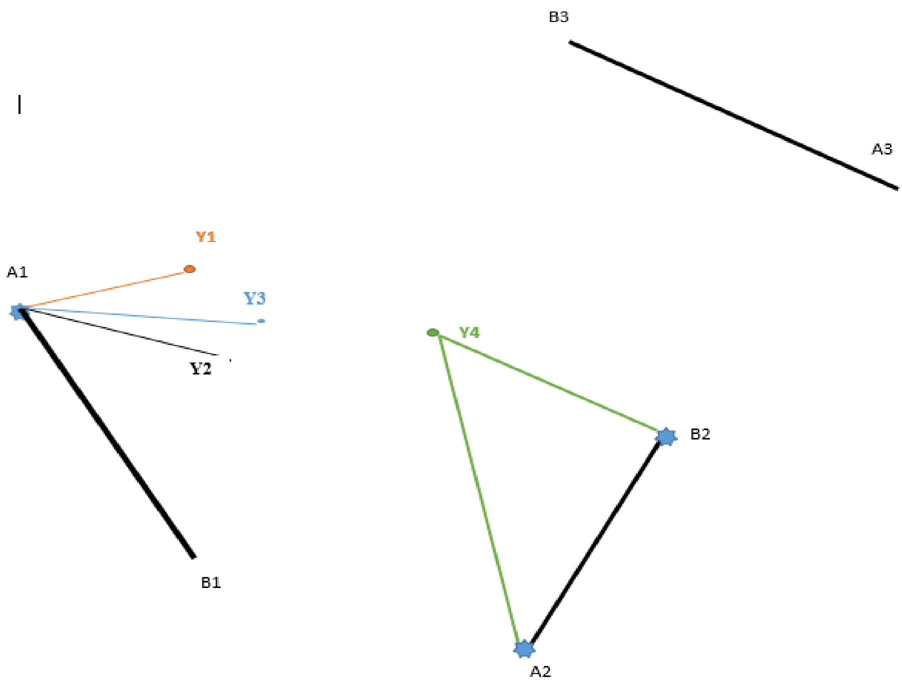

Figure 3: Concluded remarks regarding service centers

5. Conclusion and Suggested Solution for the Research Problem:

As mentioned above the problem in its original form (see [15] and [17]) did not take into account the capacity of each service center which represents the maximum number of callers it can serve simultaneously, which means that each service center is supposed to serve any number of callers at one time, this assumption is not Realistic and can cause big problems if a large number of callers need help at the same time.

The research provided a solution to the optimization problem of finding the optimal location(s) for emergency service centers, and the research included:

• Formulation of the problem assuming that each server has a certain limited capacity that represents the maximum number of callers it can serve.

• Create a polynomial algorithm to solve the problem.

• Provide a numerical example showing the steps of the algorithm.

In future researchers can investigate the following related problem:

- Determining the optimum locations or the assembly points of service in case of pandemic such situation

- Applying the solution proposed in this article and similar research in real cases in urban design in cooperation with specialists in this field.

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